The scalar spectrum in the viscous-convective subrange

By JAMES O. NYE† AND ROBERT S. BRODKEY

Department of Chemical Engineering, The Ohio State University,
Columbus, Ohio

A modified design of a fibre optic light probe was used for the measurement of turbulent concentration fluctuations and the spectra associated with these. The main object was to establish experimentally the nature of the scalar spectrum in the viscous-convective subrange. The existence of a spectral region with a (-1)-power law form supports the uniform straining model proposed by Batchelor (1959) for Schmidt numbers (ν/D) much greater than unity. For this range, the data do not agree with the cascading process suggested by Pao (1965).

An additional object was to study further the decay of the concentration fluctuations in terms of a measure of the turbulent mixing. More specifically, the limitations of applying the isotropic stationary mixing theory of Corrsin (1957, 1964a) to a shear pipe flow situation and the validity of the earlier data for this case (Lee & Brodkey 1964; Brodkey 1966a, b; and Gegner & Brodkey 1966) were investigated.

1. Introduction

It is not the purpose of this paper to develop any specific theory, but a brief historical sketch will help put the present work in perspective with that which has gone before. Furthermore, only those analyses which might be classed as intuitive and that give rise to some idea as to the shape of the scalar spectrum, will be considered. Thus, we will not discuss the various closure approximations, for which one can cite the recent article by Lee (1966) which compares a number of these for the scalar field.

1.1. *The scalar spectra*

The turbulent mixing problem was developed in the same manner as turbulent motion theory by Obukhov (1949) and Corrsin (1951a). It was shown that an inertial-convective subrange would exist in which the spectrum would have a $-\frac{5}{3}$ region

$$E_s(k) = A\epsilon_s\epsilon^{-\frac{1}{3}}k^{-\frac{5}{3}}, \tag{1}$$

where A is a dimensionless constant, ϵ_s is the rate of dissipation of concentration fluctuations, ϵ is the turbulent energy dissipation rate, and k is the wavenumber. The cut-off wave-number for this was estimated to be $k_C = (\epsilon/D^3)^{\frac{1}{4}}$. Batchelor (1959) pointed out that this should be valid if the Schmidt number is near unity, and that for systems with large Schmidt numbers (as in our case), a

† Present address: Monsanto Company, Texas City, Texas.

viscous-convective subrange should exist with a -1 spectral region. Beyond this, in a viscous-diffusive region, a rapid decay should occur. Based on his uniform straining model, he obtained for these ranges

$$E_s(k) = -(\epsilon_s/\gamma k)\exp(Dk^2/\gamma), \tag{2}$$

where γ is the strain and is approximately given by

$$\gamma = -0.5(\epsilon/\nu)^{\frac{1}{2}}. \tag{3}$$

For $k \ll (-\gamma/D)^{\frac{1}{2}}$ or $(\epsilon/\nu D^2)^{\frac{1}{4}}$, $E_s(k)$ would vary as k^{-1}. For the high Schmidt number systems, the $-\frac{5}{3}$ region would still be expected to exist for k less than
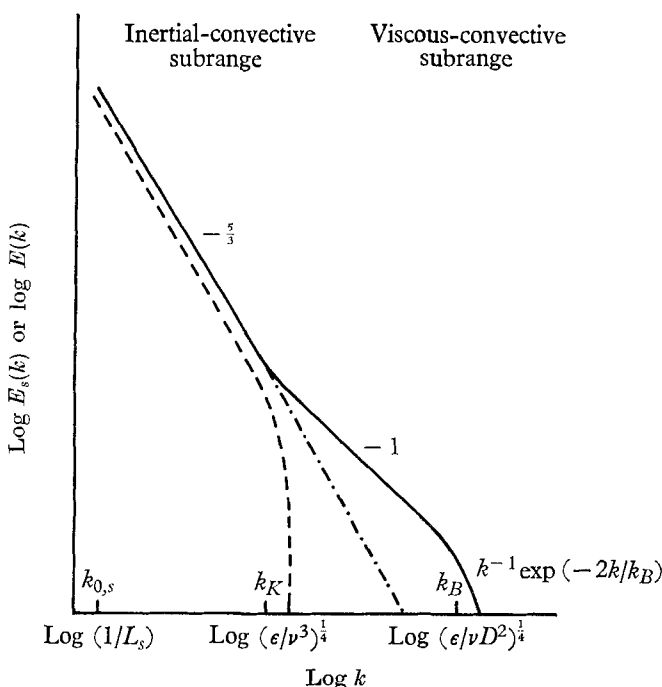

FIGURE 1. Theoretical spectra. $N_{Sc} = \mu/\rho D \gg 1$; ———, $E_s(k)$, Batchelor; —·—·—, $E_s(k)$, Pao; ————, $E(k)$.

$k_K = (\epsilon/\nu^3)^{\frac{1}{4}}$. More recently Pao (1965) has suggested that a spectral equation obtained earlier by Corrsin (1964b) for the very low Schmidt number region, should also be valid for the high Schmidt number range. The equation is

$$E_s(k) = n\epsilon_s\epsilon^{-\frac{1}{3}}k^{-\frac{5}{3}}\exp(-\tfrac{3}{2}nD\epsilon^{-\frac{1}{3}}k^{\frac{4}{3}}), \tag{4}$$

where n is a constant used as 0·59 (Gibson & Schwarz 1963b). A summary is given as figure 1.

Little experimental work has been done for high Schmidt number conditions, where the viscous-convective subrange might be observed. This involves the measurement of concentration fluctuations in liquid systems, which has proven to be most difficult (Lee & Brodkey 1963; Gibson & Schwarz 1963a; Nye & Brodkey 1967). Both the experimental results of Lee & Brodkey (1964) and

Gibson & Schwarz (1963 *b*) show a flatter than $-\frac{5}{3}$ region where the subrange would be expected to exist; however, as suggested by Pao (1965) one cannot honestly claim to have proven the existence of the region with the data available. Thus, neither the uniform straining mechanism nor the suggestion of Pao can be said to be proven. It is one object of this work to provide an answer to this problem.

1.2. *The decay of the scalar fluctuations*

With a knowledge of the shape of the scalar spectrum, one can obtain information about the scalar fluctuations, since these are an integration of the spectrum over all wave-numbers. Corrsin (1951 *b*, 1957, 1964 *a*) considered the high Schmidt number conditions for a stationary isotropic field (isotropic for both the scalar and velocity fluctuations, i.e. neither mean velocity nor mean scalar quantity was present, but only fluctuations about zero). The method paralleled the development of the analogous Kármán–Howarth equation for the velocity field; however, the continuity equation was used as the starting point instead of the motion equation. The equation in terms of correlations of the concentration fluctuations at two points in space is

$$\partial c'^2 g_s(r)/\partial t = 2c'^2 u' \left[\frac{\partial k_s(r)}{\partial r} + \frac{2k_s(r)}{r} \right] + 2Dc'^2 \left[\frac{\partial^2 g_s(r)}{\partial r^2} + \left(\frac{2}{r}\right)\left(\frac{\partial g_s(r)}{\partial r}\right) \right], \quad (5)$$

where $g_s(r)$ is the second-order correlation given by

$$g_s(r) = \overline{c(x)\,c(x+r)}/c'^2 \quad (6)$$

and $k_s(r)$ is the corresponding third-order correlation. c' is the root-mean-squared of the concentration fluctuations, and c is the instantaneous value. In the limit of zero separation of the two points, the equation reduces to

$$dc'^2/dt = -12Dc'^2/\lambda_s^2 \quad (7)$$

where, analogous to the velocity case, the microscale, λ_s, is defined as

$$\lambda_s^2 = -2/g_s''(0). \quad (8)$$

Under the assumption of constant scalar microscale, an integration of (7) gives

$$I_s = c'^2/c_0'^2 = \exp(-t/\tau), \quad (9)$$

where I_s is a measure of the turbulent mixing and the time constant is given by

$$\tau = \lambda_s^2/12D. \quad (10)$$

I_s is a very convenient measure of the degree of mixing, since it is unity for no mixing and drops to zero when the mixture is uniform. An equivalent form suggested by Danckwerts (1953) and called the intensity of segregation is

$$I_s = c'^2/\bar{c}(1-\bar{c}), \quad (11)$$

where $\bar{c}$ is the average concentration fraction of the component being mixed and $(1-\bar{c})$ would be the concentration fraction of the remaining material.

Since the time rate of change of the concentration fluctuations [as given in (7)] is also a concentration dissipation term (analogous to ϵ), it can be expressed as an

integration of the scalar dissipation spectrum, $k^2 E_s(k)$. Furthermore, the concentration fluctuation term itself is an integration of the scalar spectrum, thus, the microscale in (7) can be re-expressed as

$$\lambda_s^2 = 6 \int_0^\infty E_s(k)\,dk \bigg/ \int_0^\infty k^2 E_s(k)\,dk. \tag{12}$$

Corrsin defined the spectrum by equation (1) in the region of wave numbers bounded by $k_{0,s}$ and k_K [$k_{0,s}$ is the low wave-number limit of the $-\frac{5}{3}$ region, and k_K is the Kolmogorov wave-number given by $(\epsilon/\nu^3)^{\frac{1}{4}}$] and the k^{-1} segment of (2) from k_K out to the Batchelor cut-off given by $(\epsilon/\nu D^2)^{\frac{1}{4}}$. He used this together with (10) and (12) to arrive at an expression for the time constant of the scalar decay for the high Schmidt number conditions in a stationary isotropic turbulent field

$$\tau = \lambda_s^2/12D \simeq \tfrac{1}{2}[3(5/\pi)^{\frac{2}{3}}(L_s^2/\epsilon)^{\frac{1}{3}} + (\epsilon/\nu)^{\frac{1}{2}}\ln N_{Sc}], \tag{13}$$

where L_s is the macroscale of mixing, which is defined as the integration of the correlation given by (6) over all separation distances. Corrsin related this to $k_{0,s}$ by the expression
$$L_s = (\pi/5)k_{0,s}^{-1}.$$

A similar analysis can be done with the spectrum suggested by Pao and given by (4). Paralleling Corrsin's work and using the spectrum as a $-\frac{5}{3}$ region from $k_{0,s}$ to $k_C = (\epsilon/D^3)^{\frac{1}{4}}$, one obtains

$$\tau = 1/\epsilon^{\frac{1}{3}}k_{0,s}^{\frac{2}{3}} = (5/\pi)^{\frac{2}{3}}(L_s^2/\epsilon)^{\frac{1}{3}}, \tag{14}$$

which is independent of the diffusivity.

The combination of the preceeding equations [(9) and (13)] plus equations to allow estimations of unknown parameters has been used by Lee & Brodkey (1964), Brodkey (1966a, b) and Gegner & Brodkey (1966) to predict the decay of concentration fluctuations of an injected dye material during mixing in the turbulent pipe flow of water. A knowledge of the nature of the scalar spectrum would lend support to the application of the prediction method to practical problems of mixing. Thus the object of this work was primarily directed to the measurement of this spectrum. Since the new concentration detection system was considerably smaller than the previous one used, more reliable concentration fluctuation data could be obtained. Thus, a second object of this work was to remeasure the decay of concentration fluctuations along the centre-line during the turbulent pipe flow of water, and to check the validity of the earlier data of Lee & Brodkey (1964) as well as the applicability of the method of Corrsin to the prediction of this data. A pipe flow system was selected because the authors wished to investigate the scalar decay in a stationary turbulent field rather than the possibly more complex case of scalar decay in a decaying velocity field that would be obtained behind a grid in a water tunnel.

2. Experimental system

The flow and dye injection systems were the same as previously described by Lee & Brodkey (1964). Briefly, the unit consisted of holding tanks, a 7·8 cm polyethylene pipe test section (nominal 3-inch diameter), pump, filter, an axially

ocated dye injection tube and associated gravity feed lines, and the measuring system. The general layout is shown in figure 2. Further description and figures can be found in the reference cited. The conditions of experimentation were the same; a centre-line Reynolds number of 50,000 was used.
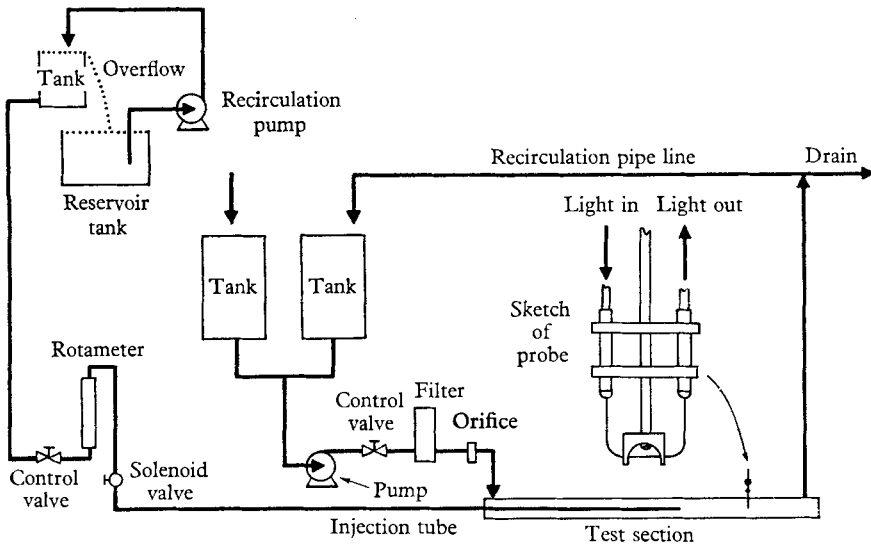

FIGURE 2. Equipment.

There are two points of difference associated with the measurements: the fibre optic light probe system and the dye used as the scalar contaminant. The old probe was described in detail by Lee & Brodkey (1963) and the newer one by Nye & Brodkey (1967). A sketch of the latter is included in figure 2. The probe has two fibre glass light conducting lines whose diameters were 0·25 and 0·38 mm. The light travelled through the larger line, crossed a 0·25 mm gap to the other line, which led to a photomultiplier tube measuring system. The new probe has a volume of approximately $1·2 \times 10^{-5}$ c.c., which is a reduction of a factor of about 45 from the previous unit. The new probe was mounted from the side wall of the pipe and located at the centre-line. There was no means provided for adjustment along the radial direction as had been done in the previous work; thus, only centre-line conditions were checked. The new dye had improved light absorption characteristics and in addition eliminated the troublesome problem of fibre optic staining. The material was surprisingly simple, being Sheaffer's no. 112 *Washable Peacock* blue ink ($D = 2·6 \times 10^{-6}$ cm^2/sec).

A high intensity projection light bulb (Sylvania DLR, 250 watts) was used to provide the light for the fibre optic probe. The fluctuating light output from the probe was picked up by a RCA IP-21 photomultiplier tube, which with its associated circuitry converted the input to a voltage. The average of this was measured with a high input impedance electrometer (Keithley 610B). The fluctuating signal was passed through a decade isolation amplifier (Keithley 102B). The r.m.s. was determined with the aid of a 9 sec time constant r.m.s. meter that was a part of a Linitronic Model 40W Linear Constant-Temperature

Hot-Film Anemometer. The signal was also recorded on tape (Ampex Instr. Tape 748 on an SP-300 recorder); however, in order to obtain better signal-to-noise ratios at the higher wave numbers, a variable filter (Allison 2) was used in some cases to cut off the lower frequency high energy fluctuation signals. Thus, the entire wave-number range could be recorded at a higher level and not be lost in tape recorder and other noise. The same spectrum analysing units were used as before, but an improved method of analysing the data was employed. Details of this will be given in the next section when the spectrum results are discussed. Other details and figures, as well as an evaluation of the modified measuring system can be found in the references cited.

3. The scalar spectra

3.1. *Experimental results*

The one-dimensional scalar spectra were measured at 4, 24 and 36 pipe diameters downstream from the dye injection point. The spectra corrected for finite probe size are shown in figure 3, as well as the uncorrected spectrum at 36 pipe diameters so as to illustrate the magnitude of the correction. The corrections were based on the work of Uberoi & Kovasznay (1951), who derived a spectrum correction factor, which was a function of $k\ell$, where ℓ is a linear dimension. Although the correction was derived for a hot-wire probe sensitive to temperature change, it should be reasonable for our cylindrical probe provided the correct value of ℓ is chosen. For this, the average of the diagonals of the inscribed and superscribed prisms as developed by Rosenwseig (1959) was used. The degree of correction shown in figure 3 was large only at high wave-numbers and even if neglected would not change the conclusions about the existence of a -1 spectral region.

A better means of correcting the data at the higher wave-numbers could be developed by rederivation of the Uberoi & Kovasznay approach for a scalar in a finite volume. However, since their correction, which is approximate for our case, did not alter the conclusion about the -1 region, we did not feel that additional effort should be spent on this aspect of the problem. If further work is to be done, it should be directed toward the development of smaller probes or completely different methods of measurement, so that corrections will not be required.

It should be pointed out that Rosensweig has developed a spectrum correction factor for concentration fluctuations in a light beam. The approach was totally different from that of Uberoi & Kovasznay, and suggested that no correction was needed below a wave-number of $100\,\mathrm{cm^{-1}}$ and would be very large at $250\,\mathrm{cm^{-1}}$. The basis for his correction is not as convincing as that for the Uberoi & Kovasznay work, and so was not used to correct the present data.

Finally, the possible effect of light scattering should be mentioned. First and foremost the probe was fixed in geometry and calibrated with known concentrations of dye. The flow system was continually filtered to eliminate solids such as fine dust, etc., thus the only scattering source would be the water and dye molecules. The effect of scattering would be in the same direction as absorption, i.e. the larger molecules would decrease the light level by scattering light out of the volume or by the main cause, absorption. In either event, the effect was accounted

for by keeping the dye concentration low so that Beer's law was obeyed and by the calibration using known dye concentrations. The spectra as measured were normalized so that their integrals would give c'^2, the mean squared concentration fluctuation, according to the relation (Hinze 1959)

$$c'^2 = \int_0^\infty \phi_s(k)\, dk. \tag{15}$$

Thus as one progresses down the pipe, the spectra are decaying.
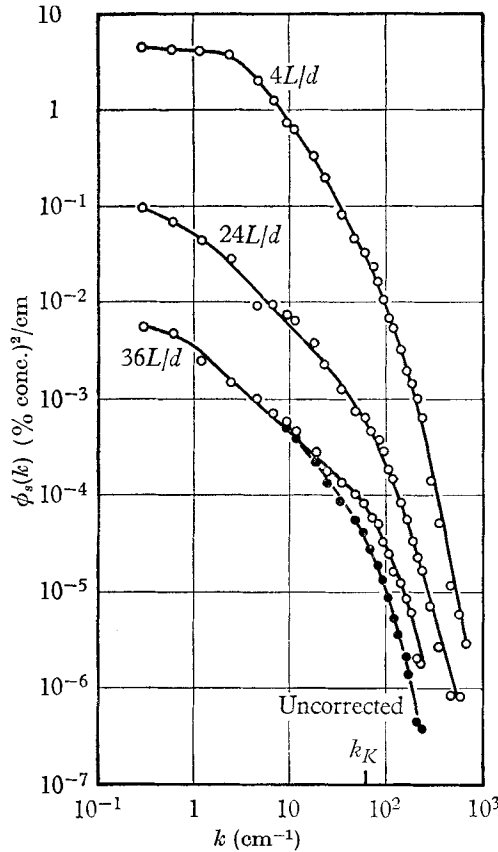

FIGURE 3. Corrected and uncorrected one-dimensional scalar spectra.

The spectra were measured on a Panoramic spectrum analyser system. A low frequency unit was used for signals below 150 Hz, with a 1 Hz bandwidth. Between 150 and 20,000 Hz, a sonic system was used in conjunction with a 10 Hz bandwidth. In each case a power Spectral Density Analyser was used, which stored the signal on an analogue integration unit (capacitor) for a period of 30 sec; the charge level was also recorded. The fluctuation signal on the tape recorded was long enough to allow at least five 30 sec periods to be studied; thus, each point of the spectrum represents an average of at least $2\frac{1}{2}$ min of sampling time. The five peaks varied about 6 % about the average value used in figure 3. It is the shape of the spectra lines that is of interest here and not their absolute magnitude.

Inadequate response of the analysing equipment below 2 Hz and errors intro-
duced because of probe resolution at the high frequency end will of course cause
errors in the shape of the lines at these extremes. In the central region the shape
will be correct, but the actual level may be wrong because of the integration of a
curve with the extremes poorly defined [in the use of (15)]. This latter error should
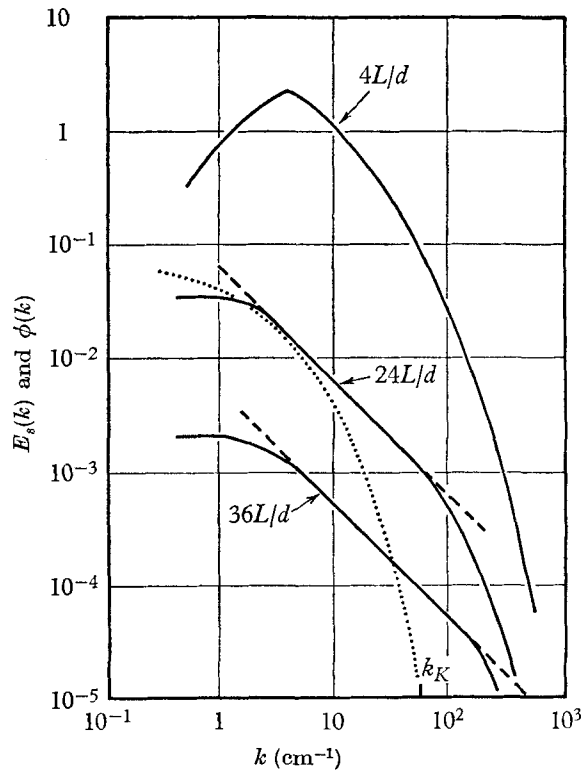not be large since the spectra curves clearly flattened out at the low wave-number

FIGURE 4. Three-dimensional scalar spectra and one-dimensional velocity spectrum.
——, $E_s(k)$ (% conc.)^2/cm; ......, $\phi(k)$ cm^3/sec^2; ————, -1.

end, which corresponded roughly to the size of the system, and could be assumed
flat to zero. At the high wave-number end, the difference between the observed
and assuming a continued -1 region was not evaluated except to note that
the contribution of this region to the integral of (15) is small. Finally, an estimate
of the effect of the correction shown in figure 3 was made. The difference in area
amounted to about $7\frac{1}{2}$%.

A simple differential relation exists between the one-dimensional and three-
dimensional spectrum for isotropic turbulence (Hinze 1959):

$$E_s(k) = -k[d\phi_s(k)/dk]. \tag{16}$$

It is a simple exercise to show that in any region where $E_s(k)$ has a constant power
dependency on k, $\phi_s(k)$ will have the same power dependency on k. Thus one would
not expect much difference in the shapes of $E_s(k)$ and $\phi_s(k)$; nevertheless, $E_s(k)$
was determined from (16) for each case and are presented in figure 4. The only

justification for the use of (16) would be the tendency towards isotropic conditions at the centreline, although in contrast to a number of claims to the contrary in the literature, the central region is most certainly not isotropic. The various theories discussed in the first section are in terms of the three-dimensional function, $E_s(k)$, and thus it is convenient to use figure 4. A comparison of figures 3 and 4 show the same general shape between $E_s(k)$ and $\phi_s(k)$, with $E_s(k)$ dropping more slowly than $\phi_s(k)$ in the rapidly decreasing regions.

3.2. *Discussion of spectra results*

For the spectrum at 36 pipe diameters as shown in figure 4, a very distinct k^{-1} region was observed as predicted by Batchelor's (1959) theory for the high Schmidt number range. For the four pipe diameters curve, a continuous decay was observed, and no segment could be said to have a -1 slope. At 24 diameters, a -1 region of slightly less than one decade was obtained, while at 36 diameters, the spectrum showed about $1\frac{1}{2}$ decades. The -1 region would normally be expected to follow a $-\frac{5}{3}$ segment; however, for this to exist there must be a corresponding region in the velocity spectrum. This spectrum as shown on figure 4 had no such region due to a lack of an inertial subrange and even a lack of isotropic conditions in the pipe flow shear field. For this system, Lee & Brodkey (1964) determined k_K as $62\,\mathrm{cm}^{-1}$ from velocity spectrum measurements. Experimentally, the $-\frac{5}{3}$ region ends at about $0\cdot05k_K$, which would correspond to a k of about $3\cdot2\,\mathrm{cm}^{-1}$ for the present system. The scale of such a wave-number range is about $0\cdot3\,\mathrm{cm}$ and is in the range of the flow system dimensions (a radius of about $4\,\mathrm{cm}$); thus, the conditions of an inertial subrange would not be expected to exist, since such eddies would most certainly obtain a reasonable part of the energy from the gross system. It should be emphasized that, as Batchelor pointed out, the existence of the -1 region does not depend on the existence of the $-\frac{5}{3}$ inertial subrange. However, the estimate of the strain as given by (3) would probably be incorrect.

The general shape of the spectrum at 36 pipe diameters from the injector was in agreement with the predictions of Batchelor (1959); however, the position of the spectrum with respect to the expected wave-number was displaced. Batchelor suggested that the -1 region should follow the $-\frac{5}{3}$ segment and that the change-over point would be around k_K. More recent analysis and data indicate the end of the $-\frac{5}{3}$ inertial subrange of the velocity spectrum to occur about $0\cdot05k_K$. Thus, one might modify Batchelor's change-over prediction somewhat. The present results support this in that the -1 region started at about $6\,\mathrm{cm}^{-1}$, which is $0\cdot1k_K$, and is in good agreement with the expected end of the $-\frac{5}{3}$ segment had it existed. The -1 region of the spectrum ended much sooner than predicted by Batchelor's theory. This was attributed to the probe response rather than to any deficiency in the theory. The probe did not give a true picture of the eddies in the very high wave-number region where they would be much smaller than the probe itself.

Pao's theory as given by (4) and based on the idea of a scalar spectral cascading rate dependent only on ϵ and k, predicts a $-\frac{5}{3}$ region throughout the wave-numbers encountered in this work. This was not observed, so that one must

question the applicability of (4) to the high Schmidt number range. This does not detract from its possible validity at much lower Schmidt or Prandtl numbers.

In the limit of zero diffusivity (infinite Schmidt number) there is no mixing as measured by a decrease in the concentration fluctuations. For these conditions, the time constant obtained from Corrsin's analysis, which uses the $-\frac{5}{3}$ and -1 regions as given by (1) and (2), is infinite. This result allows no decrease in c' as is logically correct. In contrast, the result obtained from the extended $-\frac{5}{3}$ region suggested by Pao's equation (4) is independent of the diffusivity and gives finite mixing in the limit. Actually, (14) is identical to the equation obtained by Corrsin (1957, 1964a) for the case where the Schmidt number is unity, which again suggests that Pao's analysis is not valid for the high Schmidt number range.

4. The decay of the scalar fluctuations along the centre-line

Lee & Brodkey (1964) measured the mean concentration profiles and found them to have a bell shape which spread progressively toward the wall with decreasing height, as in a jet. Since the pipe wall was impermeable, the profiles became flattened as the mean concentration profile approached the final uniform concentration at the downstream end. The intensity of the concentration fluctuations was measured simultaneously with the mean concentration profiles. Examples of both profiles can be found in the reference cited and in the recent review article by Brodkey (1966a). The intensity of concentration fluctuations remained highest along the centre-line and thus was used by Lee & Brodkey as an upper bound for the decay of intensity in the whole region of the pipe. Later, Gegner & Brodkey (1966) used Lee's data to establish the mixing characteristics at all radial positions rather than just at the centre-line. The work reported here is restricted to the centre-line and was obtained to test the validity of the earlier data. In addition, the new data could be used for a better test of applying the isotropic stationary mixing theory of Corrsin (1957, 1964a) to a shear pipe flow situation. It should be emphasized that if a reasonable check is obtained it does not imply that the flow is isotropic but rather constitutes a use of isotropic theory, not a proof.

The decay of the concentration fluctuations along the centre-line of the pipe is quite rapid and is a result of two contributing factors; the decay due to mixing is superimposed on the decay of the mean as a result of the spread of the injected material away from the centre-line. Since it is the true decay due to mixing, in the absence of decay of the mean that is desired, the 'intensity of segregation' as first defined by Danckwerts (1953) was calculated from the data in exactly the same manner as suggested by Lee & Brodkey (1964). Figure 5 shows the present experimental results and the results of Lee & Brodkey (1964) as corrected by Brodkey (1966b). Although the two mixing systems were identical, the probe used by Lee & Brodkey was larger and apparently was not adequately sensitive to the small-scale fluctuations. The new decay does not follow an exact straight line, and as pointed out by Gegner & Brodkey (1966), the least-squares fit they used is really only an average value. In theirs and more pronounced in the present case, the experimentally observed slope of the decay curve increased as one moved away from the injector.

The distance decay of figure 5 is also shown as a time decay. This is done by using the length travelled by a blob of fluid divided by the local mean velocity, which in this case is the maximum velocity at the centre-line; i.e.

$$t = x/\overline{U}_{\text{max}}. \tag{17}$$

For the present experimental results, the time constant of the decay is about 1·7 sec for the initial period, and decreases to about 0·8 sec far from the injector. The corresponding results for the older data are 1·1 and 0·8 sec. The least mean squares line for the older data as obtained by Gegner & Brodkey (1966) corresponded to a time constant of 1·03 sec, a result which is quite close to that obtained
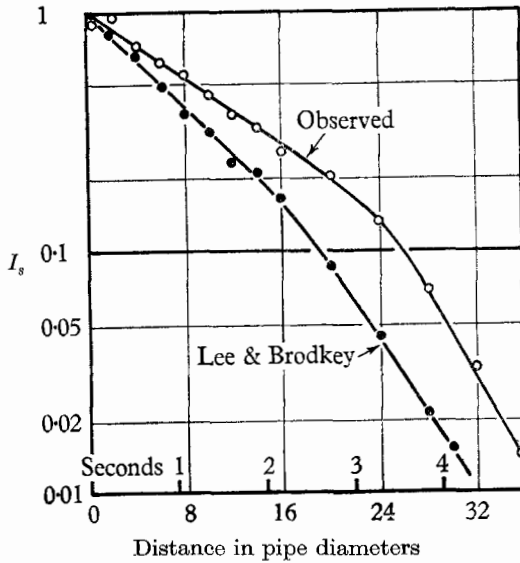

FIGURE 5. Decay of the intensity of segregation.

for the present data but clearly not as meaningful. For these same conditions, the time constant estimated from (13) by Gegner & Brodkey was 0·93 sec. The value predicted is still quite close to that experimentally observed but clearly not constant as implied from the theory. Nevertheless, the theory is adequate for a reasonable estimation of the time constant of mixing.

The fact that the decay curve is not linear warrants further comment. Equations (12) and (13) establish the dependence of the time constant on the spectrum measurements presented in the previous section. First, it is clear that the integrations involved in developing Corrsin's theory have a smoothing effect so that information about the details of spectrum shape are lost. The fact that the experimental spectra do not have a $-\frac{5}{3}$ region does not appear important for the estimation. Apparently the area under the theoretical curve used by Corrsin and the experimental curve are close so that the comparison is good. More important, however, is the expected shape of the decay curve as implied from the actual spectra measurements. From figure 4, one can see that the spectra have not reached a steady state as far as their shape is concerned, and thus by (12) the

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microscale cannot be constant. The level of the spectrum is not important, since any constant would cancel out, but the position with respect to the wave-number is. From figure 4, spectra gradually shift to the right as the steady state far down the pipe is approached. As the shift occurs the values of

$$\int_0^\infty k^2 E_s(k)\,dk$$

would be larger with respect to

$$\int_0^\infty E_s(k)\,dk$$

and cause a decrease in the microscale and hence in the value of the time constant. Thus, as observed, one would expect an increasing slope of the decay curve as the steady state spectrum is approached.

As pointed out on several previous occasions, the macroscale of mixing (L_s), which is key in (13), has never been directly measured. Indirect estimates of this are used as described by Gegner & Brodkey. Although the equipment is not as yet available, the author hopes to be able to make autocorrelations of the recorded turbulent concentration fluctuation data at the three measuring points. In this manner a direct measurement of L_s could be obtained, and used to check the reliability of the currently used estimates of this.

5. Conclusions

The concentration spectrum has a k^{-1} region that in the present work extended for $1\frac{1}{2}$ decades. Its position with respect to wave-number was such that it began shortly after the $k^{-\frac{5}{3}}$ region should have ended, if it had existed in our experiments. Thus, most of the details of Batchelor's theory are confirmed. The concentration spectra extended far beyond the velocity spectrum as predicted and reported earlier by Lee & Brodkey. The concentration spectrum cut-off after the -1 region occurred before it was expected to do so, but this is attributed to limited probe response rather than to a deficiency in Batchelor's theory. A rough estimate based on decay measurements indicate that the cut-off was in accord with the theory, but this could not be checked with the present experimental probe system. A $-\frac{5}{3}$ region did not exist over the wave-numbers investigated as predicted by Pao's theory.

The size of the probe clearly affects the results of the decay and spectrum measurements. Probe correction are reported in more detail by Nye & Brodkey (1967) and show that these should be small for the decay results of the present work; however, as indicated above, a high wave-number attenuation occurred in the spectrum measurements.

The decay of the intensity of segregation follows roughly the exponential form as predicted by Corrsin. However, since the microscale of mixing was changing as the spectrum approached a steady state, some curvature was observed in the decay curve as would be expected. Corrsin's analysis was based on the steady state that was never reached in these experiments. Nevertheless, the time

constant as predicted by the theory is quite close to the range observed and provides a means of estimating turbulent mixing.

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